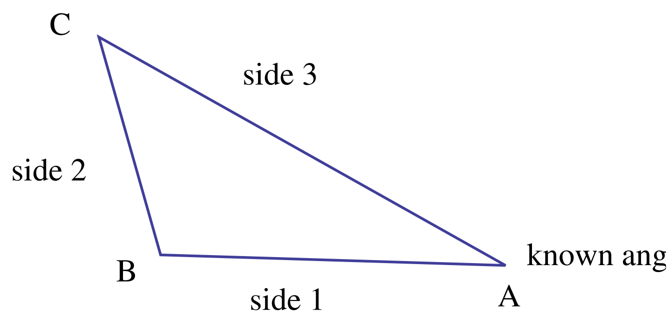
[11-03-04-T]

The law of sines, all cases

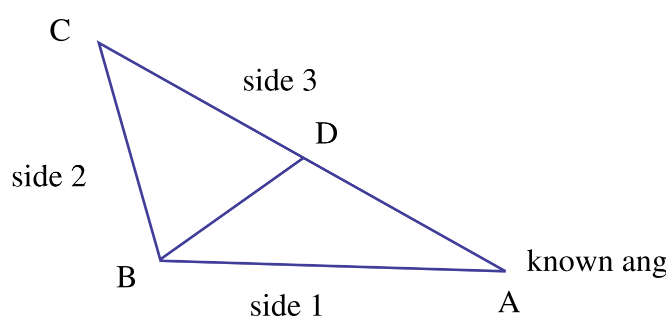
1. Draw this triangle. Place known angle at right vertex, then place the other known quantities.



Note that if only side-2 and side-3, there is no chance of using Law of Sines. If side-1 and side-3 are given, then exactly one triangle is possible, because SAS. If side-1 and side-2 given, this is SSA so there may be 0, 1, or 2 triangles consistent with the given information.

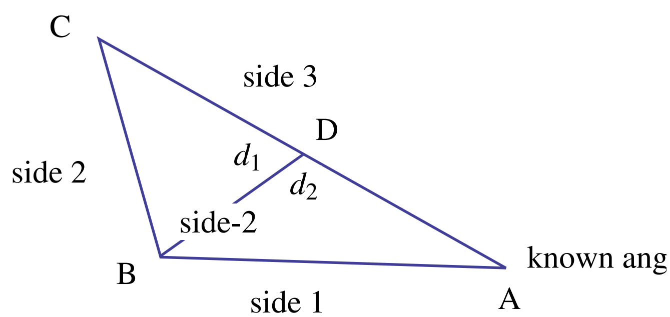
2. Solve triangle ABC, realizing that △ABC exists iff $A + B + C = 180\,°$.

3. $A + (180 - C) < 180\,°$ iff there exists one more triangle, △ABD, consistent with the given information.



Angle D of △ABD must be equal to $180\,° - C$. Since you know angle A, angle D, and side-1, it is a simple matter to solve triangle ABD.

You should be able to explain why it is true that $A + (180 - C) < 180°$ iff there exists one more triangle, △ABD.



Since BC = BD, △BDC isosceles. So, $C = d_1$. Since d_1 and d_2 supplementary, $d_2 = 180 - C$. Triangle ABD exists just in case $A + B + d_2 = 180°$. Since B is positive, $A + d_2 < 180°$. Or, equivalently, $A + (180 - C) < 180°$.

We could take this a little further by noting that $A + (180 - C) < 180 \Longleftrightarrow A - C < 180 \Longleftrightarrow A < C$. So our condition for △ABD to exist is $A < C$.